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Heat/mass transfer and dynamics of bubbles in high-polymer solutions—II. Oscillations in a sound field

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Abstract—Heat/mass transfer and the dynamics of bubbles in a high-polymer solution exposed to ultrasonic agitation are investigated. An exact solution of the problem is found on the basis of which main features of the oscillations of bubbles are analysed with account taken of the relaxational properties of the carrying phase.

INTRODUCTION

IN THE first part of this work [1], the analysis of free oscillations of bubbles in polymer solutions due, in particular, to a pulsed pressure change in the surrounding medium was conducted. In what follows, oscillations and heat/mass transfer of bubbles in a sound field are studied within the scope of a similar approach. The solution of the problem stated is obtained with the use of a system of transfer equations and boundary conditions formulated in Part I. The equations and symbols listed in ref. [1] are not given here.

GENERAL SOLUTION

Consider the interaction of a longitudinal wave in a hereditary polymer medium with a single bubble containing a mixture of an inert gas and the liquid phase vapour. The long wave approximation typical of the acoustics of two-phase media [2] will be employed which assumes the fulfilment of the condition $l_2' \gg R_0$. This condition imposes the limitation on the sound wave frequency $f \ll c_2 R_0^{-1}$, which, at $c_2 \sim 10^3$ m s^{-1}, yields the estimate for the limiting frequency $f_{\max} \sim 10^5$ Hz for $R_0 \sim 10^{-2}$ m. The approximation of pressure homogeneity in the bubble ($l_1 \gg R_0$), which is assumed in what follows, is stronger, since the speed of sound in the vapour–gas phase is much smaller than that in the liquid. When $c_1 \sim 10^2$ m s^{-1}, $f_{\max} \sim 10^4$ Hz for $R_0 \sim 10^{-2}$ m. Since with a decrease in R_0 the value of $f_{\max}$ increases, the given approximation is admissible over a wide range of frequencies of the sound field and sizes of bubbles [3].

Let $\{p_a^*, \theta_a^*\} \exp(i\omega\tau)$ denote dimensionless perturbations of pressure and temperature in a falling wave on the bubble surface (here and hereafter the same characteristic quantities for non-dimensionalization are employed as those in ref. [1]). To determine the spherical scattered wave, we construct the solution of a linearized system of transfer equations in a hereditary medium (equations (1), (2), (6)–(8) from ref. [1]). The dimensionless perturbations of velocity, pressure, density and temperature in the scattered wave are sought respectively in the form $\{\tilde{v}_2, \tilde{p}_2, \tilde{\rho}_2, \tilde{\theta}_2\} \exp(i\omega\tau)$. After the determination of the unknown amplitude functions, the velocity, temperature and pressure fields in the vicinity of the oscillating interphase boundary are determined by superimposing the characteristics of the incident and scattered waves. This yields

$$v_2^* = (v_{21}^* + v_{22}^*)\,e^{i\omega\tau}, \quad \theta_2^* = (\theta_a^* + \theta_{21}^* + \theta_{22}^*)\,e^{i\omega\tau},$$

$$p_2^* = (p_a^* + p_{21}^* + p_{22}^*)\,e^{i\omega\tau},$$

$$v_{2k}^* = -\xi^{-2}(1 + im_k\xi)A_k\,e^{-im_k\xi},$$

$$p_{2k}^* = -(i\omega\xi)^{-1}[4/3(G_1^* + i\omega\eta_s^*)m_k^2 - \omega^2]A_k\,e^{-im_k\xi},$$

$$\theta_{2k}^* = v_k v_5 \xi^{-1} A_k\,e^{-im_k\xi}, \quad k = 1, 2. \qquad (1)$$

Here $m_{1,2}$ are dispersion equation roots differing in modulus:

$$am^4 - bm^2 - c = 0$$

$$a = G_5^*, \quad b = \omega^2 - (i\omega G_3^{*2} Pe_2)/c_{2P}^* - i\omega G_4^* G_5^* Pe_2 \Gamma_2^{-1},$$

$$c = i\omega^3 Pe_2 G_4^* \Gamma_2^{-1}, \quad G_1^* = \tilde{G}_1^* p_{20}^{-1},$$

$$G_2^* = K_1^* + \tilde{G}_2^* p_{20}^{-1}, \quad G_3^* = \alpha^* K_1^* + \tilde{G}_3^* T_0 p_{20}^{-1},$$

$$G_4^* = 1 - \tilde{G}_4^*(\rho_{20} c_{2V})^{-1},$$

NOMENCLATURE

c_k	speed of sound
Im	imaginary part
j	complex dimensionless amplitude of phase changes
l_k	length of sonic wave ($k = 1, 2$)
m_σ	mass of surfactant
P	complex dimensionless amplitude of pressure in a bubble
p_a	pressure amplitude in a falling wave
Re	real part.

Greek symbols

θ_a	amplitude of temperature change in a falling wave
τ	dimensionless time, t/t_0
ω	dimensionless frequency of acoustic pressure, $\tilde{\omega}t_0$.

Subscripts

1	vapour–gas mixture
2	liquid
r	value of parameter in resonance.

Superscript

*	dimensionless value; complex dynamic modulus.

$$G_5^* = i\omega\eta_v^* + G_2^* + 4/3(G_1^* + i\omega\eta_s^*),$$
$$v_{1,2} = m_{1,2}^2 - \omega^2 G_5^{*-1}, \quad v_{3,4} = 1 + im_{1,2},$$
$$v_5 = iG_5^*(\omega G_3^*)^{-1}. \tag{2}$$

The dimensional complex dynamic moduli of the hereditary liquid $\tilde{G}_i^*$ ($i = 1, 2, 3, 4$) and also the dynamic coefficients of viscosity $\tilde{\eta}_k^*(k = 1, 2)$ that characterize the rheological properties of the medium in the case of periodic deformation with frequency $\tilde{\omega}$ are defined by the relations

$$\tilde{G}_i^* = \tilde{G}_i' + i\tilde{G}_i'' = \int_0^\infty F_i(\lambda)(\tilde{\omega}\lambda) \times (\tilde{\omega}\lambda + i)[1 + (\tilde{\omega}\lambda)^2]^{-1}\,d\lambda$$
$$\tilde{\eta}_k^* = \tilde{\eta}_k' - i\tilde{\eta}_k'' = \tilde{G}_k^*/i\tilde{\omega}.$$

Equation (2) determines the complex wave numbers of the coupled sonic (m_1) and thermal (m_2) perturbations. The expressions for m_1 at $G_4^* = 1$ in the approximation of weak absorption of a sound wave have the form

$$m_1 = \omega K_S^{*-1/2}[1 - 1/2K_S^{*-1}(4/3G_1' + G_2')] - 1/2i\omega^2 K_S^{*-3/2}[4/3(\eta_1' + \eta_S^*) + \eta_2' + \eta_v^* + Pe_2^{-1}(\Gamma_2 - 1)],$$
$$K_S^* = \Gamma_2 K_1^*, \quad G_{1,2} = \mathrm{Re}\{G_{1,2}^*\}, \quad \eta_{1,2}' = \mathrm{Re}\{\eta_{1,2}^*\}. \tag{3}$$

According to experimental data, within the range of frequencies $f \ll 1$ MHz for polymer solutions of moderate concentrations, the dispersion of sound (the possibility of which is indicated by relation (2)) is usually small [4] which is due to the smallness of the moduli $G_{1,2}'$. When $\eta_{1,2}' = G_{1,2}' = 0$, equations (3) characterize the speed and decay of the sonic wave in a viscous fluid with account taken of the volumetric viscosity η_v and heat absorption. In the case of a viscous fluid, the expression for m_2 takes the form $m_2^2 = -i\omega Pe_2\Gamma_2^{-1}$. Calculations by equation (2) show that in a relaxing medium, just as in a viscous fluid, the thermal mode decays rapidly. It will be assumed in what follows that the source of thermal perturbations is absent or is located rather far from the inclusion and therefore the discussion will be restricted to the study of the interaction of the bubble only with the sonic mode. Then, the amplitudes of temperature and pressure perturbations in the falling wave turn out to be connected by the relation

$$\theta_a^* = -i\omega v_1 v_5[4/3(G_1^* + i\omega\eta_S^*)m_1^2 - \omega^2]^{-1} p_a^*.$$

It should be noted that for a spherical wave scattered by the bubble the conservation of the thermal mode in the solution is fundamental in view of the necessity to fulfil the boundary conditions.

The radius of the bubble oscillating in a sound field is sought in the form of the real part of the expression $R^* = 1 + \delta\exp(i\omega\tau)$, $|\delta| \ll 1$. The solution of the linearized system of equations of transfer in the vapour–gas phase and the determination of the integration constants from boundary conditions is carried out similarly to the case of natural oscillations. This yields the expressions for the amplitudes of oscillations of bubble radius, pressure P in the mixture and intensity of phase transitions j:

$$\delta = (D_1 Y_2 - D_3 Y_1)\Delta_1^{-1}, \quad P = (D_4 Y_1 - D_2 Y_2)\Delta_1^{-1},$$
$$j = v_7(\alpha_6 Q_4 p_{10}^*)^{-1}\{Q_5 P + p_{10}^* Q_8[v_8(\theta_a^* - P_{10}^{*-1}(\alpha_4 + v_6 Q_5)P) + v_9\omega\delta]\},$$
$$\Delta_1 = D_1 D_4 - D_2 D_3. \tag{4}$$

Here α_1–α_7, Q_1–Q_5 and Ψ are determined from relations given in ref. [1]. The expressions for α_8, α_9 and Q_6–Q_{11} are derived from similar formulae of ref. [1] after the substitution $h \to i\omega$. The combinations of the parameters $v_k(k = 6, 7, \ldots, 10)$ coincide with n_k with the only difference being that now $m_{1,2}$ values are determined according to equation (2). The remaining

dimensionless quantities are given below:

$$D_1 = \beta_3^2 - 3\alpha_8[1+\Gamma_1(\alpha_4-1)] + v_6^{-1}\Psi(\alpha_4 - Q_{10}),$$

$$D_2 = p_{10}^*(3\Gamma_1\beta_3^2 - i\omega v_6^{-1}Q_9\Psi),$$

$$D_3 = 1 + (v_5 p_{10}^*)^{-1}\omega(m_2-m_1)(\alpha_4 + v_6 Q_5)Q_8,$$

$$D_4 = 2\sigma^* + i\omega Q_8\{i\omega(Q_6-Q_7) + 4(i\omega)^{-1}(G_\infty^* + i\omega\eta_S^*)(v_4Q_6 - v_3Q_7)\},$$

$$\beta_1^2 = i\omega\, Pe_D, \quad \beta_3^2 = i\omega\, Pe_1,$$

$$Y_1 = p_{10}^* v_6^{-1}\Psi(1+Q_{11})\theta_a^*,$$

$$Y_2 = \omega v_5^{-1}(m_2-m_1)Q_8\theta_a^* + p_a^*.$$

In the case of an incompressible liquid phase ($v_1 = v_5 v_1 = m_1 = 0$, $v_5 = \infty$) at $G_4^* = 1$, solution (4) is greatly simplified to

$$\delta = p_a^*(D_4 - D_2/D_1)^{-1}, \quad P = p_a^* - D_4\delta,$$

$$j = v_7 D_5^{-1}(p_{10}^{*-1}v_{10}P - v_4\theta_a^*), \quad D_2 = 3p_{10}^*\beta_3^2\Gamma_1,$$

$$D_1 = \beta_3^2 - 3\alpha_8[1+\Gamma_1(\alpha_4-1)] + v_{10}(v_4v_6-1)^{-1}\Psi,$$

$$D_4 = 2\sigma^* + \omega^2 - 4(G_\infty^* + i\omega\eta_S^*), \quad m_2 = -i\beta_2. \tag{5}$$

From equation (5) it follows that a vapour–gas bubble in an incompressible fluid can be considered as an oscillator with effective dissipation coefficient v and elastic constant β:

$$v = 2\eta_S^* + 2\omega^{-1}\,\mathrm{Im}\{G_\infty^*\} + (2\omega)^{-1}\,\mathrm{Im}\{D_2/D_1\},$$

$$\beta = 4\,\mathrm{Re}\{G_\infty^*\} + \mathrm{Re}\{D_2/D_1\} - 2\sigma^*. \tag{6}$$

For gas bubbles ($k_0 = 0$, $\kappa = 0$) oscillating in an incompressible viscous fluid ($G_\infty^* = 0$) relations (6) coincide with the results of ref. [5]. Solution (4) for a viscous fluid coincides with the corresponding relation in ref. [6], accurate to characteristic time scale.

NUMERICAL RESULTS AND DISCUSSION

Calculations of the oscillations of bubbles in a relaxing fluid exposed to a sound field were carried out on an example of polystyrene solution in toluene. The parameters of the system, the technique of calculation and the results are presented in the same way as in ref. [1]. The data given in Figs. 1–5 were obtained from formulae (5). For the plots of Figs. 1 and 2 the frequency of the sound field was $f = 18$ kHz. The influence of the fluid compressibility was investigated on the basis of general solution (4). This influence is discussed separately.

The amplitude of gas bubble pulsations is characterized by the plots of Fig. 1. They show that the inclusion of viscoelasticity effects leads to fundamental changes in the pattern of oscillations of inclusions in a sound field.

Calculation based on the rheological model of a Newtonian fluid yields strongly damped oscillations due to the high viscosity of the system, and the resonance is virtually absent. However, the account for even one relaxation time sharply varies the character of oscillations (curve 1). When n_1 increases, the resonance amplitude decreases and when $n_1 = n_\infty^*$ the limiting curve is attained. The value of n_∞^* decreases with decreasing frequency f and an increasing spectral distribution parameter z_1. The effect of the latter is essential only in the zone of resonance and is more pronounced for the frequencies $f < f_r$ or for inclusions with $R_0 < R_r$. When the viscoelastic properties of the fluid are taken into account, the phase of oscillations increases in the pre-resonance zone and decreases for $f > f_r$.

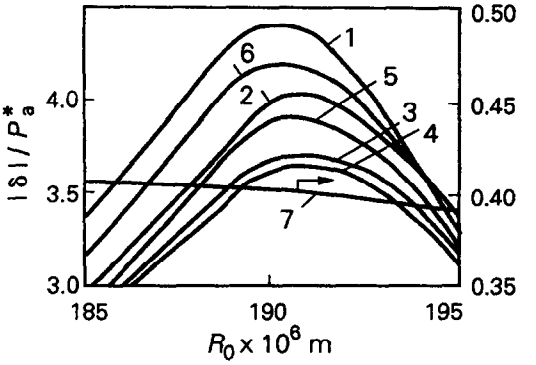

FIG. 1. Effect of the relaxation spectrum on the relative amplitude of gas bubbles. $T_0 = 293$ K. 1–4, $z_1 = 2$, $n_1 = 1$, 50, 200, n_∞^*; 5, 6, $n_1 = n_\infty^*$, $z_1 = 2.3, 3$; 7, a Newtonian fluid with $\eta_p = \eta_S = 0.5$ Pa s.

The results of calculations of the dynamics and heat and mass transfer of bubbles in a polymer solution at high temperatures are characterized by the plots of Figs. 2–5. When the system is heated, the concentration of vapour in bubbles grows and the res-

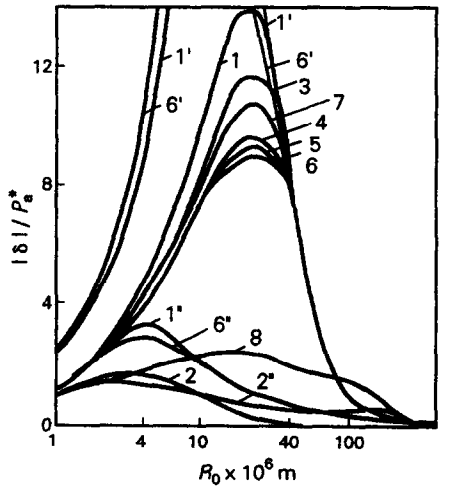

FIG. 2. The relative amplitude of oscillations of vapour–gas and vapour bubbles in a viscoelastic polymer solution. $T_0 = 383.7$ K. $a = 1$; 1″, 2″, 6″, IT; 1′, 6′, ICC; 1, 1′, 1″ and 2, 2″, a Newtonian fluid with $\eta_p = \eta_S = 5 \times 10^{-4}$ and 0.117 Pa s, respectively; 1, 1′, 2–6, 6′, 7, $\kappa = \infty$; 1″, 2″, 6″, $\kappa = 0.02$; 3–6, 6′, 6″, $z_1 = 2$; 7, $z_1 = 2.5$; 3–5, $n_1 = 10$, 50, 100; 6, 6′, 6″, 7, $n_1 = n_\infty^*$. $a = 0.9$; 8, IT; $\kappa = \infty$; $z_1 = 2$, $n_1 = n_\infty^*$.

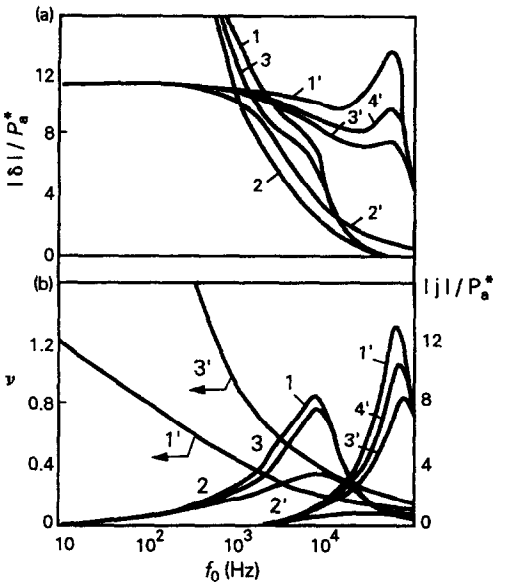

FIG. 3. Characteristics of vapour–gas bubbles pulsating in a viscoelastic fluid. $T_0 = 383.7$ K; $\kappa = \infty$; IT. 1–3, $R_0 = 10^{-4}$ m; 1′–4′, $R_0 = 10^{-5}$ m; 1, 1′, $\eta_p = \eta_S = 5 \times 10^{-4}$ Pa s; 2, 2′, $\eta_p = \eta_S = 0.117$ Pa s; 3, 3′, $z_1 = 2$; $n_1 = n_1^*$; 4, 4′, $z_1 = 2.5$; $n_1 = n_1^*$.

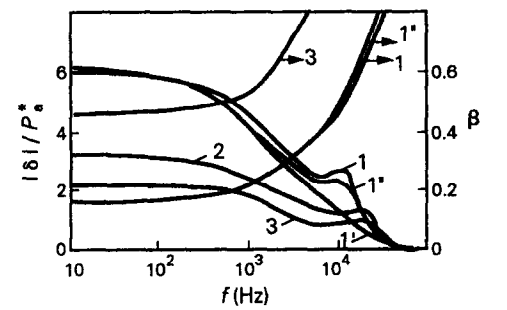

FIG. 4. Effect of activity coefficient magnitude. $T_0 = 383.7$ K; IT; $\kappa = \infty$, $R_0 = 10^{-4}$ m; 1, 1′, 1″, $a = 0.95$; 2, 3, $a = 0.9$, 0.85; 1″, 2, 3, $z_1 = 2$, $n_1 = n_1^*$; 1, 1′, $\eta_p = \eta_S = 0.5 \times 10^{-3}$, 0.117 Pa s.

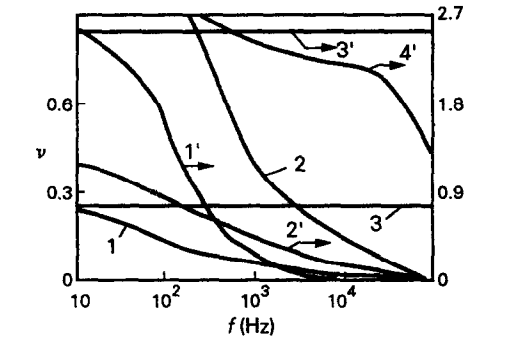

FIG. 5. Rheological and thermal coefficients of dissipation. $T_0 = 383.7$ K; IT. 1–3, $R_0 = 10^{-4}$ m; 1′–4′, $R_0 = 10^{-5}$ m; 1, 1′, v_r; 2, 2′, v_t; 4′, v; 3, 3′, viscous dissipation coefficient for a Newtonian fluid at $\eta_p = \eta_S = 0.117$ Pa s; 1, 1′, 2, 2′, 4′, $z_1 = 2$, $n_1 = n_1^*$; for all the curves, except 4′, $\kappa = \infty$; 4′, $\kappa = 0.02$.

onance curve shifts to the zone of small inclusions. As is seen from Fig. 2, for $T_0 = T_b$ the effect of the fluid viscoelasticity, which leads to an increase in the resonance amplitude and also in the intensity of phase transitions, is especially high in the case of quasi-equilibrium character of the processes of evaporation and condensation. The increase in the number of relaxation times in the spectrum, taken into account in calculations, leads to a decrease in the value of $|j|$ and in the resonance amplitude (curves 3–6). The growth of the spectral distribution parameter z_1 exerts the opposite effect (curves 6 and 7).

It follows from Fig. 3(a) that for $T_0 = T_b$ in the region of low frequencies of the sound field with $f \to 0$ the pulsation amplitude of a bubble with $R_0 = 10^{-4}$ m increases quickly (linear solution of equation (4) becomes incorrect in this case), whereas for an inclusion with $R_0 = 10^{-5}$ m the amplitude remains limited. This difference is attributable to different vapour concentrations in bubbles. In fact, when $\omega \to 0$ there exists [6] a critical concentration of vapour k_* (defined by the same expression as in the case of free oscillations [1]), at which $\lim_{\omega \to 0} \delta = \infty$. This result is explained physically by the vanishing of the effective elasticity of the bubble β in the low-frequency region at $k_0 = k_*$ when an insufficient self-elasticity of the vapour–gas mixture is 'quenched' by the negative contribution of capillary pressure. For $k_0 > k_*$ with $\omega \to 0$ we have $\beta < 0$, which corresponds to an unstable oscillating system. As applied to the solution considered with $R_0 = 10$ and 100 μm, $k_* = 0.985$ and 0.998, respectively, whereas the equilibrium vapour concentration in these bubbles at $T_0 = T_b$ constitutes $k_0 = 0.957$ and 0.995. It is the closeness of the latter value of k_0 to k_* which explains the growth of the oscillation amplitude of the bubble with $R_0 = 100$ μm when $\omega \to 0$.

The listed values of k_* are higher than for water [6], which is due to the smallness of the surface tension coefficients characteristic for many volatile organic fluids. For this reason, it is essential to take into account the effect of concentration of polymer in a solution on the pressure of saturated vapours, since for $a < 1$ the amplitude δ turns out to be limited for all f values under the conditions considered and at the same temperature (Fig. 4).

The dissipation coefficient v for bubbles in a polymer solution decreases with decreasing frequency (curves 3′ in Fig. 3(b) and 4′ in Fig. 5), approaching at large f values the value corresponding to a pure solvent. Therefore, the influence of the relaxation properties of a solution turns out to be especially important for inclusions, the resonance frequencies of which are high. It follows from equation (6) that the parameter v can be represented as a sum of the rheological, v_r, and thermal, v_t, coefficients of dissipation:

$$v_r = 2\eta_S^* + 2\omega^2 \operatorname{Im}\{G_1^*\}, \quad v_t = (2\omega)^{-1} \operatorname{Im}\{D_2/D_1\}. \tag{7}$$

The behaviour of each of them is characterized by the plots of Fig. 5. It is seen that the frequency increases, both the thermal and rheological losses decrease simultaneously. Comparison of curves 1, 1′ and 2, 2′ shows that with a decrease in R_0 the role of rheological dissipation increases, while that of thermal dissipation decreases. It should be noted that with the non-equilibrium phase transition the thermal dissipation is much higher than in the case of quasi-equilibrium bubbles, due to which the effect of the viscoelasticity of fluid for quasi-equilibrium inclusions is much more pronounced than for non-equilibrium bubbles.

In order to investigate the role of compressibility of the hereditary medium, a number of calculations were performed on the basis of full solution (4). The results indicated that the influence of acoustic losses, which lead to a smaller resonance amplitude of inclusions, is predominantly manifested for gas bubbles with $R_0 \geqslant 10^{-3}$ m and can approximately be taken into account by the acoustic dissipation coefficient $\nu_a = \omega^2/(2c_2^*)$, $c_2^* = c_2/v_0$. In this case the role of the bubble fluid viscoelasticity effects is negligibly small for both vapour–gas and gas inclusions. This coincides with the conclusion made in ref. [7] within the scope of a simpler rheological model.

SURFACE EFFECTS

Analysis of the surface activity effects in interphase interaction dynamics is of great interest, since many polymers in solutions, just like low-molecular surfactants, can greatly alter the surface tension. The role of non-solvable surfactant film in oscillations of a gas bubble was studied in ref. [8]. Using a similar approach, it is also possible to estimate, in the first approximation, the influence of capillary effects on oscillations of vapour–gas inclusions in the systems considered.

Let the surface concentration of the surfactant be $\Gamma = m_\sigma/4\pi R^2$ with $\sigma = \sigma(\Gamma)$ (it is assumed that $m_\sigma = \text{const.}$). Suppose that the uniform distribution of the surfactant over the bubble surface is preserved in the presence of radial oscillations. Then, for the case of small oscillations

$$\sigma = \sigma_0[1-\varepsilon_\sigma(R_0^2R^{-2}-1)], \quad \varepsilon_\sigma = g\Gamma_0/\sigma_0,$$
$$\sigma_0 = \sigma(\Gamma_0), \quad \Gamma_0 = m_\sigma/4\pi R_0^2,$$
$$g = -(\partial\sigma/\partial\Gamma)_{\Gamma=\Gamma_0} > 0. \tag{8}$$

According to equation (8), the quantity σ enters the dynamic boundary condition on the bubble surface [1]. As a result, in solution (4) the quantity σ^* in D_4 is replaced by $\sigma_0^*(1-2\varepsilon_\sigma)$. In the remaining expressions σ^* should also be understood as σ_0^*.

Analysis of the relations obtained in this way with regard to the considered polymer solution in toluene showed that the presence of the surfactant leads to the damping of oscillations of small vapour–gas inclusions of pre-resonance size with $k_0 < k_*$ (note that a decrease in the static coefficient of surface tension in the solution is responsible for the growth of k_*). The effect is associated with an increase in the bubble elasticity coefficient β which, in the presence of a surfactant, is equal to

$$\beta = 4\,\mathrm{Re}\{G_1^*\} + \mathrm{Re}\{D_2/D_1\} - 2\sigma_0^*(1-2\varepsilon_\sigma).$$

It is of interest to note that for purely vapour micro-inclusions the effect of the surfactant reverses, since the increase in the magnitude of β is accompanied in this case by a decrease in $|\beta|$. It should be borne in mind, however, that free oscillations of bubbles, which are characterized by negative effective elasticity, are unstable in amplitude [1, 9].

CONCLUSIONS

The effect of the relaxational properties of a polymer fluid leads to a substantial decrease in rheological dissipation losses in the course of the oscillations of inclusions in a sound field, thus leading to a sharp increase in the resonance amplitude of oscillations of constant mass bubbles as against a similar viscous fluid. This result explains, in particular, the experimentally observable fact [10] of the development of acoustic cavitation and oscillations of bubbles in a polymer melt with a very high Newtonian viscosity $\eta_p \sim 10^5$ Pa s. In the case of vapour–gas bubbles, the relaxation effects are responsible for the growth of the oscillation amplitude near the resonance, which is especially important for quasi-equilibrium inclusions. In the case of non-equilibrium character of the evaporation–condensation processes, the influence of the viscoelasticity effects decreases.

The fall of the solvent vapour pressure over the solution leads to a decrease in the oscillation amplitude of inclusions at a fixed temperature $T_0 \sim T_b$. This factor is especially important in the low-frequency zone where there is a critical concentration of vapour in bubbles which leads to a drastic increase in the amplitude of oscillations with $f \to 0$.

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TRANSFERT THERMIQUE/MASSIQUE ET DYNAMIQUE DES BULLES DANS DES SOLUTIONS DE HAUTS POLYMERES—II. OSCILLATIONS DANS UN CHAMP SONORE

Résumé—On étudie le transfert de chaleur/masse et la dynamique des bulles dans une solution de haut polymère exposée à l'agitation ultrasonique. Une solution exacte du problème est trouvée et les configurations principales des oscillations des bulles sont analysées en tenant compte des propriétés relaxationnelles de la phase vectrice.

WÄRME- UND STOFFÜBERTRAGUNG UND BLASENDYNAMIK IN HOCHPOLYMEREN LÖSUNGEN—II. OSZILLATIONEN IN EINEM SCHALLFELD

Zusammenfassung—Wärme- und Stoffübertragung sowie die Blasendynamik in einer hochpolymeren Lösung, die einer Ultraschallerregung ausgesetzt ist, werden untersucht. Eine exakte Lösung des Problems wird ermittelt, in dem die Hauptmerkmale der Blasenschwingungen unter Berücksichtigung der Relaxationseigenschaften der Trägerphase signalisiert werden.

ТЕПЛОМАССООБМЕН И ДИНАМИКА ПУЗЫРЬКОВ В РАСТВОРАХ ВЫСОКОПОЛИМЕРОВ—II. ПУЛЬСАЦИИ В ЗВУКОВОМ ПОЛЕ

Аннотация—Исследуется тепломассообмен и динамика пузырьков в растворе высокополимера при ультразвуковом воздействии. Найдено точное решение задачи, на основе которого в широком температурном диапазоне проанализированы основные особенности пульсаций пузырьков с учетом релаксационных свойств несущей фазы.